Determining the Functional Parameters of a Simple Speed Regulator

Sorin VLASE
Transylvania University of Brașov, B-dul Eroilor,29, Romania, svlase@unitbv.ro

Mircea MIHĂLCICĂ
Transylvania University of Brașov, B-dul Eroilor,29, Romania, mihalcica.mircea@unitbv.ro

Maria Luminița SCUTARU
Transylvania University of Brașov, B-dul Eroilor,29, Romania, lscutaru@unitbv.ro

Abstract: - The paper makes an analysis of a speed regulator for a uniform response using some mechanism with periodic motion. For the considered mechanism with one degree of freedom the conditions for the quasi-uniform motion are computed, in the case where the vibrating mass is a rigid coupling with the elastic and damping forces. The stability of the solutions in the phase space and the limit cycles for a uniform response of the system is established.

Keywords: - vibration, regulator, phase space, damping

1. INTRODUCTION

Applications of regulators are frequently found in engineering practice. The problem is to find, for each type of application, the most suitable controller for the intended purpose. In almost all cases, the mathematical model leads us to non-linear dynamic systems, and so the study is done by numerical or approximate methods, drawing on the known results from the stability analysis in the linear case. The paper aims to determine the functional parameters of a regulator by averaging non-linear, non-stationary terms in dynamic systems. Each constructive solution may have advantages in terms of energy dissipation but also disadvantage due to the practical realization and cost of the proposed solution. Using this method is possible to obtain the functional parameters in an analytical manner. The motion of the system is necessary to be uniform and, with this consideration in mind, the conditions that the mechanical and geometric parameters have to fulfill will be determined. It is a major advantage that the differential equations of the system can be solved only numerically and can not draw qualitative conclusions quickly.

To find solutions - similar to the method of variation of constants, we consider that coefficients and parameters vary slowly, adapting to our problem the Van der Pol's method [8]. In the paper, we propose a solution characterized by its engineering simplicity. The analytical solution obtained using an averaging method of the differential equations is verified by numerical procedures. In this way, the achieved degree of confidence in the regulator increases and allows for the achievement of a trustworthy solution that corresponds to the proposed goal.

2. STUDY OF A REGULATOR WITH A SLIDER CRANK MECHANISM

Let us consider the mechanism in Figure 1 where we have: r – radius of the crank, l – the length of the rod, $\lambda = r/l$. The mechanism is driven by a constant motor torque. In the absence of any friction the system will move faster, so the velocity will always increase. The problem is to find the mechanical parameters of this system so that, under the action of the constant torque, the rotation speed of the steering wheel is also considered to be on average constant, ie the movement is stabilized. The links between the elements are holonomic. If we note ω_1 and ε_1 respectively as the angular velocity and the angular acceleration of the flywheel with the center in O, and ω_2 and ε_2 as the angular velocity and the angular acceleration of the rod AB, we have:

$$\dot{\varphi} = \omega_1; \quad \dot{\beta} = -\omega_2; \quad \dot{\omega}_1 = \varepsilon_1; \quad \dot{\omega}_2 = \varepsilon_2. \qquad (1)$$

Then the coordinate of the position vector of the center of mass C of the homogeneous bar can be expressed:

$$x_C = r\cos\varphi - \frac{l}{2}\sin\beta; \quad y_C = r\sin\varphi - \frac{l}{2}\cos\beta; \qquad (2)$$

For point B we can write:

$$y_B = r\sin\varphi - l\cos\beta\,. \tag{3}$$

Also, in the triangle OAB we have:

$$r\cos\varphi = l\sin\beta \tag{4}$$

If we differentiate (4) we find:

$$-r\dot{\varphi}\sin\varphi = l\dot{\beta}\cos\beta \tag{5}$$

or:

$$-r\omega_1\sin\varphi = -l\omega_2\cos\beta\,, \tag{5'}$$

and yet:

$$\omega_2 = \frac{r\sin\varphi}{l\cos\beta}\omega_1 = t\omega_1 \tag{6}$$

where:

$$t = \frac{r\sin\varphi}{l\cos\beta} = \lambda\frac{\sin\varphi}{\cos\beta} \tag{7}$$

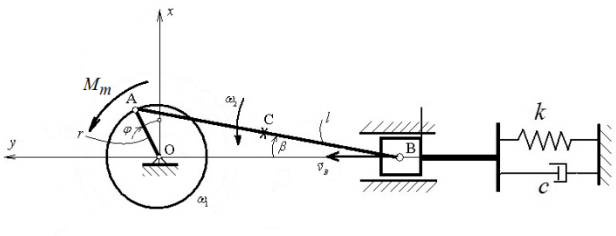

Figure 1. Regulator mechanism

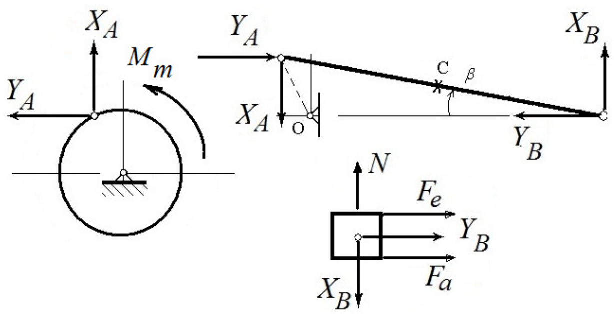

Figure 2. Free body diagram

By differentiating again (5') we find:

$$-r\varepsilon_1\sin\varphi - r\omega_1^2\cos\varphi = -l\varepsilon_2\cos\beta - l\omega_2^2\sin\beta \tag{8}$$

so that we have:

$$\varepsilon_2 = \frac{r\sin\varphi}{l\cos\beta}\varepsilon_1 + \left(\frac{r\cos\varphi}{l\cos\beta} - t^2\frac{\sin\beta}{\cos\beta}\right)\omega_1^2 = t\varepsilon_1 + u\omega_1^2 \tag{9}$$

with:

$$u = \lambda\frac{\cos\varphi}{\cos\beta} - t^2\frac{\sin\beta}{\cos\beta}\quad. \tag{10}$$

We can write that:

$$\begin{Bmatrix}\omega_1\\ \dot{x}_C\\ \dot{y}_C\\ \omega_2\\ \dot{y}_B\end{Bmatrix} = \begin{Bmatrix}1\\ -r\sin\varphi + \frac{l}{2}t\cos\beta\\ r\cos\varphi - \frac{l}{2}t\sin\beta\\ t\\ r\cos\varphi - lt\sin\beta\end{Bmatrix}\omega_1 = \{A_1\}\omega_1 \tag{11}$$

and after differentiation we get:

$$\begin{Bmatrix}\varepsilon_1\\ \ddot{x}_C\\ \ddot{y}_C\\ \varepsilon_2\\ \ddot{y}_B\end{Bmatrix} = \begin{Bmatrix}1\\ -r\sin\varphi + \frac{l}{2}t\cos\beta\\ r\cos\varphi - \frac{l}{2}t\sin\beta\\ t\\ r\cos\varphi - lt\sin\beta\end{Bmatrix}\varepsilon_1 +$$

$$+\begin{Bmatrix}0\\ -r\cos\varphi + \frac{l}{2}t^2\sin\beta + \frac{l}{2}u\cos\beta\\ -r\sin\varphi + \frac{l}{2}t^2\cos\beta - \frac{l}{2}u\sin\beta\\ u\\ -r\sin\varphi + lt^2\cos\beta - lu\sin\beta\end{Bmatrix}\omega_1^2 =$$

$$= \{A_1\}\varepsilon_1 + \{A_2\}\omega_1^2 \tag{12}$$

where notations for $\{A_1\}$ și $\{A_2\}$ are obvious.

If fundamental theorems are applied [5]-[7], [9], [10], considering the mechanism as being composed of three rigid bodies, it will be possible to write:

- For the drum, we can write the theorem of the kinetic moment in O:

$$J_O\varepsilon_1 = M_m - X_A r\sin\varphi + Y_A r\cos\varphi\,.$$

- For the AB bar, after applying the impulse theorem and the kinetic momentum theorem, we obtain:

$$m_b\ddot{x}_C = -X_A + X_B$$
$$m_b\ddot{y}_C = -Y_A + Y_B$$
$$J_C\varepsilon_2 = X_B\frac{l}{2}\cos\beta - Y_B\frac{l}{2}\sin\beta + X_A\frac{l}{2}\cos\beta - Y_A\frac{l}{2}\sin\beta$$

- The slider C will have a rectilinear translation motion, so it will be possible to write:

$$m\ddot{x}_B = -F_a - F_e - Y_C$$

where:

$$F_a = c\,\dot{y}_B = c\omega_1\left(r\cos\varphi - lt\sin\beta\right)$$

$$F_e = k(y_B - y_o) = k(l + r\sin\varphi - l\cos\beta)\ .$$

Therefore we have the motion equations of the system:

$$\begin{bmatrix} J & & & & \\ & m_b & & & \\ & & m_b & & \\ & & & J_C & \\ & & & & m \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \ddot{x}_C \\ \ddot{y}_C \\ \varepsilon_2 \\ \ddot{y}_B \end{Bmatrix} =$$

$$= \begin{Bmatrix} M_m - X_A r\sin\varphi + Y_A r\cos\varphi \\ -X_A + X_B \\ -Y_A + Y_B \\ X_B\frac{l}{2}\cos\beta - Y_B\frac{l}{2}\sin\beta + X_A\frac{l}{2}\cos\beta - Y_A\frac{l}{2}\sin\beta \\ -F_a - F_e - Y_B \end{Bmatrix} \qquad (13)$$

or, if we consider the kinematic conditions:

$$\begin{bmatrix} J & & & & \\ & m_b & & & \\ & & m_b & & \\ & & & J_C & \\ & & & & m \end{bmatrix}\left(\begin{Bmatrix} 1 \\ -r\sin\varphi + \frac{l}{2}t\cos\beta \\ r\cos\varphi - \frac{l}{2}t\sin\beta \\ t \\ r\cos\varphi - lt\sin\beta \end{Bmatrix}\varepsilon_1 + \right.$$

$$\left.\begin{Bmatrix} 0 \\ -r\cos\varphi + \frac{l}{2}t^2\sin\beta + \frac{l}{2}u\cos\beta \\ -r\sin\varphi + \frac{l}{2}t^2\cos\beta - \frac{l}{2}u\sin\beta \\ u \\ -r\sin\varphi + lt^2\cos\beta - lu\sin\beta \end{Bmatrix}\omega_1^2\right) = \begin{Bmatrix} M_m \\ 0 \\ 0 \\ 0 \\ -F_a - F_e \end{Bmatrix} +$$

$$+\begin{bmatrix} -r\sin\varphi & r\cos\varphi & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ \frac{l\cos\beta}{2} & -\frac{l\sin\beta}{2} & \frac{l\cos\beta}{2} & -\frac{l\sin\beta}{2} \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{Bmatrix} X_A \\ Y_A \\ X_B \\ Y_B \end{Bmatrix} \qquad (14)$$

Using the notations above, we can group the equations as:

$$\begin{aligned} &[m]\left(\{A_1\}\varepsilon_1 + \{A_2\}\omega_1^2\right) = \left\{Q^{ext}\right\} + \left\{Q^{leg}\right\} = \\ &= \left\{Q^{ext}\right\} + [N]\{R\} \end{aligned} \qquad (15)$$

The mechanical work of the liaison forces can be written [10]-[14]:

$$\begin{aligned} dL &= \{\delta\Delta\}^T\{Q\}^{leg} = \{\dot{\Delta}\}^T\{Q\}^{leg}dt = \\ &= \{\dot{q}\}^T[A_1]^T\{Q\}^{leg}dt = \omega_1\{A_1\}\{Q\}^{leg}dt = 0 \end{aligned}$$

from where we have:

$$\{A_1\}^T\{Q\}^{leg} = 0 \qquad (16)$$

The vector $\left\{Q^{leg}\right\}$ represents the torsor of the generalized liaison forces corresponding to the generalized coordinates considered. If we multiply the equation (5) with $\{A_1\}^T$ we obtain the motion equation:

$$J(\varphi)\ddot{\varphi} + \frac{1}{2}J'(\varphi)\dot{\varphi}^2 = M(\varphi) \qquad (17)$$

where:

$$J(\varphi) = \{A_1\}^T[m]\{A_1\};\ J'(\varphi) = \{A_1\}^T[m]\{A_2\} \quad (18)$$

$$\begin{aligned} M(\varphi) &= M_m - \left(r\cos\varphi - lt\sin\beta\right)(F_a + F_e) = \\ &= M_m - \left(r\cos\varphi - lt\sin\beta\right)\left[c\omega_1\left(r\cos\varphi - lt\sin\beta\right) + \right. \\ &\left. + k(l + r\sin\varphi - l\cos\beta)\ \right] = M_m - kl\left(r\cos\varphi - lt\sin\beta\right) - \\ &- k\left(r\cos\varphi - lt\sin\beta\right)\left(r\sin\varphi - l\cos\beta\right) \end{aligned} \qquad (19)$$

3. STABILIZED SOLUTION

In the following, we will average the equation in order to obtain a stabilized solution (using van der Pol method [8]). We calculate:

$$\overline{J'(\varphi)} = \frac{1}{2\pi}\int_0^{2\pi}\{A_1\}^T[m]\{A_2\}d\varphi = 0$$

$$\overline{M_m} = M_m\ ;\quad \overline{k\left(r\cos\varphi - lt\sin\beta\right)(r + l)} = 0\ ;$$

$$\overline{k\left(r\cos\varphi - lt\sin\beta\right)\left(-r\sin\varphi + l\cos\beta\right)} = 0\,;$$

$$\overline{c\omega_1\left(r\cos\varphi - lt\sin\beta\right)^2} = cl^2\omega_1(0{,}5\lambda^2 + 0{,}000204)$$

because:

$$\overline{\cos^2\varphi} = 0{,}5\ ;\quad \overline{t^2\sin^2\beta} = 0{,}000204\ ;$$

$$\overline{t\cos\varphi\sin}\,\beta=0$$

We obtain the angular velocity which ensures the quasi-uniform rotation of the system, from the equation:

$$M_m = cl^2\omega_1(0{,}5\lambda^2+0{,}000204) \qquad (20)$$

from which:

$$\omega_1 = \frac{M_m}{cl^2(0{,}5\lambda^2+0{,}000204)}\,. \qquad (21)$$

So we have a linear relationship between the engine's movement and the stabilized velocity of the system.

For the system's operating speed, the energy input in the engine moment system is dissipated by the friction occurring in the damper.

To verify the obtained relationship (21), a numerical integration of the equation (17) will be made.

For this, after making the notations: $x_1=\alpha$; $x_2=\omega_1$, the equation becomes the linear system:

$$\begin{cases}\dot{x}_1 = x_2 \quad ; \\ \dot{x}_2 = \dfrac{M(x_1)}{Je(x_1)} - \dfrac{1}{2}\dfrac{Jp(x_1)}{Je(x_1)}x_2^2 \quad ;\end{cases}$$

In order to solve it, we use Matlab's solvers based on the Runge-Kutta method.

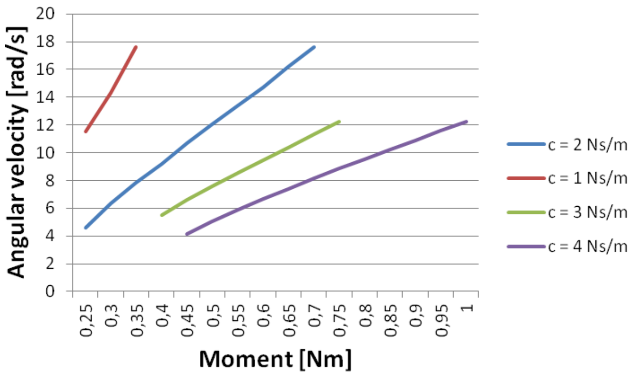

Figure 3. Variation of the angular velocity for different damping coefficients

In Figure 3 we present the angular-moment velocity diagram for different damping coefficients. The linear shape of the angular moment-velocity dependence is observed.

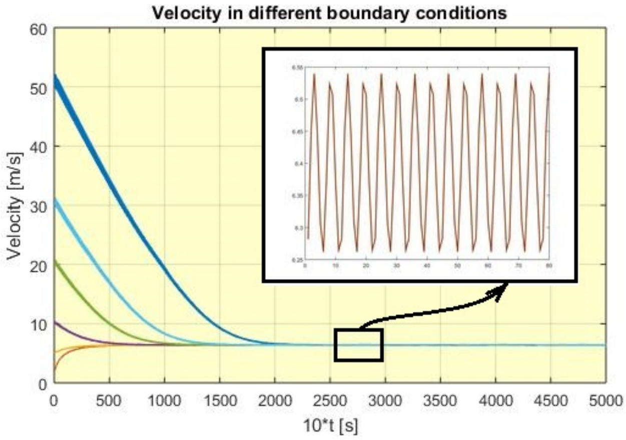

Figure 4. Stabilization of angular velocity for different initial conditions: ω_{st} = 5.07 m/s

Figure 4 shows the angular velocity graphs for different initial conditions. It is noted that the angular velocity stabilizes for the chosen mechanical parameters at ω = 5.07 rad/s. The variation of the angular velocity for different damping coefficients (with the same initial conditions) is shown in Figure 5.

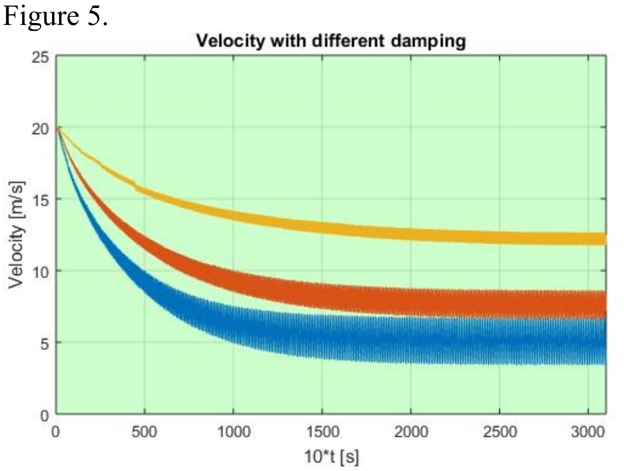

Figure 5. Variation of angular velocity for different damping (same initial conditions- c=2; c=3; c=4 Ns/m)

If the system motion is braked (energy dissipated through the damper is higher than the energy input by the motor moment), the damped motion is shown in Figure 6.

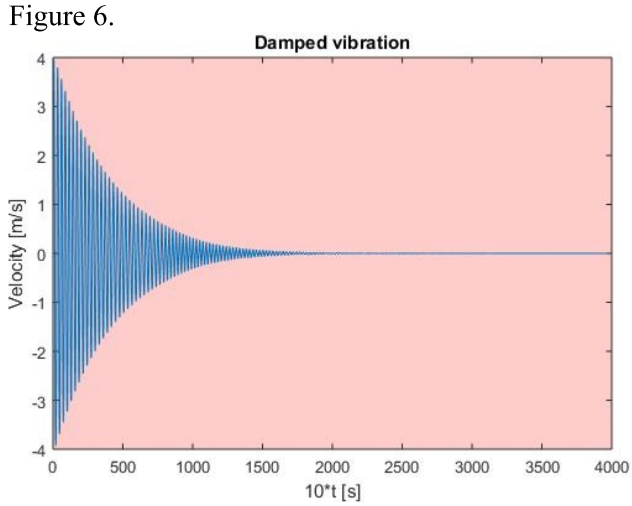

Figure 6. Damping system's vibrations

A representation in the phase space of the piston trajectories is shown in Figure 7.

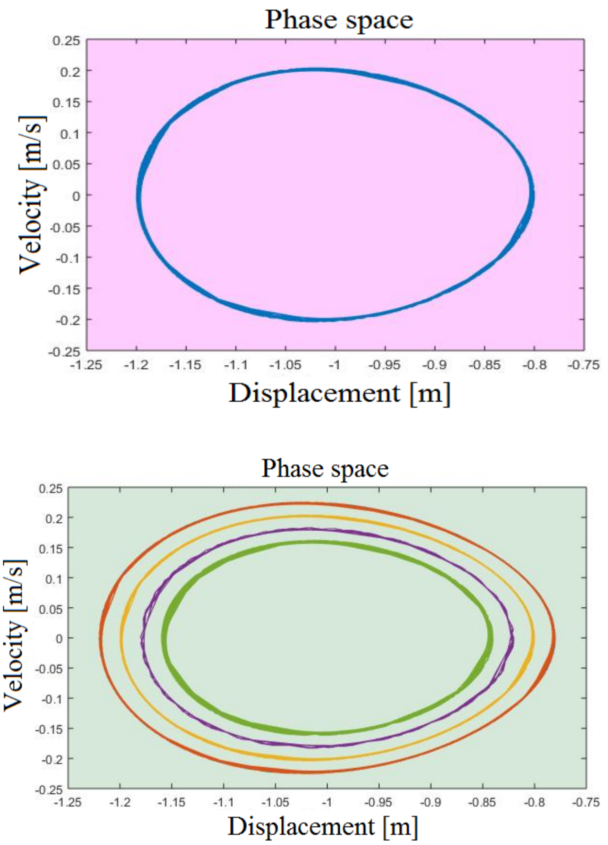

Figure 7. Representing trajectories in phase space

4. CONCLUSIONS

In the paper we study a speed regulator for uniform regime, in the case of non-autonomous vibratory mechanisms. The purpose of the proposed regulator is to provide a quasi-uniform rotation of a wheel of a device. The energy introduced through the external moment will be dissipated, in a stationary regime, through the damper used. In order to do this, it is necessary to calculate the characteristics of the damper, a difficult thing to do generally because of the need to integrate a system of nonlinear differential equations. The quasi-uniform motion conditions are deducted if the vibrating mass is rigidly coupled to the elastic and damping forces according to the crank rod model. Considering the nonlinear equations, we apply the Van der Pol's Mediation and Variance Conversion method. The stability of phased space solutions, the limit cycles for uniform rotation of the flywheels and the resonance conditions are studied. This method allows the determination of the functional parameters of the system that provide a quasi-uniform motion with an imposed angular velocity, by a simple and easy-to-use method. If these parameters are known, then it is possible to make a right choice check by performing the numerical integration of the motion equations. In this way, the confidence in the design of the regulator and in the system increases.

REFERENCES

[1] Bratu P., Elastic systems vibrations. Technical Publishing House, Bucharest,2000

[2] Bratu P., Vasile O., Modal analysis of the viaducts supported on the elastomeric insulators within the Bechtel constructive solution for the Transilvania Highway. *Romanian Journal of Acoustics and vibration 9 (2), 77-82 (2012)*

[3] Bratu P., Dragan N., Vasile O., Experimental studies of sound absorption coefficient of composite materials used for acoustic treatments of the cabins, *The 11-th International Congress on Automotive and Transport Engineering CONAT, 177-181* (2010)

[4] Negrean I., Advanced notions in Analytical Dynamics of Systems. *Acta Technica Napocensis - Applied Mathematics, Mechanics and Engineering, Vol. 60, Nr.4 (2017)*

[5] Negrean I., Advanced equations in Analytical Dynamics of Systems. *Acta Technica Napocensis - Applied Mathematics, Mechanics and Engineering, Vol. 60, Nr.4 (2017)*

[6] Pennestri' E., de Falco D., Vita L., An Investigation of the Inuence of Pseudoinverse Matrix Calculations on Multibody Dynamics by Means of the Udwadia-Kalaba Formulation, *Journal of Aerospace Engineering, Volume 22, Issue 4, pp. 365-37 (2009)*

[7] van der Pol B., A theory of the amplitude of free and forced triode vibrations, *Radio Review, 1 701-710 (1920)*

[8] Vlase S., Dynamical Response of a Multibody System with Flexible Elements with a General Three Dimensional Motion, *Romanian Journal of Physics, Volume 57, Number 3-4, pp.676–693 (2012)*

[9] Vlase S., Teodorescu, PP., Elasto-dynamics of a solid with a general "rigid" motion using FEM model. Part I.*Rom. Journ. Phys., Vol. 58, Nos. 7–8, P. 872-881, Bucharest, (2013).*

[10] Vlase S., Danasel, C, Scutaru, ML, Mihalcica, M, Finite element analysis of two-dimensional linear elastic systems with a plane "Rigid motion", *Romanian Journal of Physics, Vol 59, IS 5-6, p.476-487 (2014)*

[11] Vlase S., Teodorescu, PP, Itu, C, Scutaru, ML, Elasto-dynamics of a solid with a general "rigid" motion using FEM model. Part II. Analysis of a double cardan joint. *Romanian Journal of Physics. Vol 58, IS 7-8,p882-892 (2013).*

[12] Vlase S., Dynamical Response of a Multibody System with Flexible Element with a general Three-Dimensional Motion. *Romanian Journal of Physics, Vol. 57, IS 3-4, p676-693 (2012)*

[13] Vlase S., Marin M., Öchsner A. et al. Motion equation for a flexible one-dimensional element used in the dynamical analysis of a multibody system. *Continuum Mech. Thermodyn. (2018), pp.1-10.* https://doi.org/10.1007/s00161-018-0722-y